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On Almost Feebly Totally Continuous Functions in Topological Spaces

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Abstract: **The purpose of this paper is to introduce and study the class of feebly totally open (resp. feebly totally closed), almost feebly totally open (resp. almost feebly totally closed) functions. Furthermore some of their properties are analysed. The purpose of this paper is to formulate and establish some results on almost feebly open mappings.**

Keywords: **Feebly totally open, feebly totally closed, almost feebly totally open, and almost feebly totally closed.**

1. INTRODUCTION

In 1982, S.N. Maheswari and P.C.Jain defined and studied the concepts of feebly open and feebly closed sets in topological spaces [5]. In this concepts, some new class of functions and related results were introduced and investigated namely feebly continuous [2], feebly normality, feebly regularity [4]. We recall that in the subset A of X, the feebly closure of A is the intersection of all feebly closed sets containing A and the feebly interior of A is the union of all feebly open sets contained in A, denoted by f.cl(A) and f.int(A) respectively. In 1987, Maio and Noiri [6] defined a subset A of X to be semi-regular open if A=sInt(sCl(A)) and also defined on other hand, a subset A of a space (X,τ) is said to be semi regular open if both semi open and semi closed. Throughout this paper (X,τ) or simply X denote a topological space with topologyτ, unless otherwise stated explicity.

2. PRELIMINARIES

Let us recall the definitions and results which are used in the sequel

Definition 2.1[5]: A subset A of a topological space (X, τ) is said to be feebly open (resp. feebly closed) if $A\subset sCl(Int(A))$ (resp. $s\,Int(Cl(A)) \subset A$).

Definition 2.2[3]: Let (X, τ) be a topological space. Any subset A of X is called feebly clopen if it is both feebly open and feebly closed.

Definition 2.3: A map $f : X\to Y$ is said to be:

i. Feebly closed (resp. feebly open) [7] if the image of each closed set (resp. open set) in X is feebly closed (resp. feebly open) set in Y.
ii. Feebly continuous [7] if $f^{-1}(V)$ is feebly open in X for each open set V of Y.
iii. Feebly clopen [3] if the image of every open and closed set in X is both feebly open and feebly closed in Y.
iv. Totally feebly continuous [3] if the inverse image of every open subset of (Y,σ) is a feebly clopen subset of (X, τ).
v. Slightly feebly continuous [3] if the inverse image of every feebly clopen set in Y is feebly open in X.
vi. Feebly clopen irresolute [3] if $f^{-1}(V)$ is feebly clopen in (X, τ) for each feebly clopen set V of (Y,σ).

Remark 2.4:

i. Every open set is feebly open [1].

ii. Every closed set is feebly closed [1].

iii. Every feebly clopen mapping is feebly closed map and feebly open map [3].

iv. A mapping $f : X \to Y$ is feebly clopen if f satisfies the conditions $f(H^o) \subset (f(H))^o$ for every $H \subset X$ and $\overline{f(H)} \subset f(\overline{H})$ for every $H \subset X$ [3].

3. FEEBLY REGULAR OPEN SETS

In this section, a new class of feebly regular open sets and feebly regular open mappings, are discussed.

We recall the following definitions.

Definition 3.1: A subset A of X is said to be feebly regular open (briefly F.reg.open) if A = f.int (f.cl(A)).

Remark 3.2: The feebly regular open set is analysed in the way if A is both feebly open and feebly closed.

Definition 3.3: A subset A of X is said to be feebly regularly closed if A = f.cl (f.int(A)) (briefly F.reg.closed).

Definition 3.4: A subset A of X is said to be feebly regular clopen if A= f.int (f.cl(f.int(A))). On the other hand, if A is F.reg.open and F.reg.closed.

Definition 3.5: Let A be subset of X. The feebly regular closure of A (briefly F.reg.cl(A)) is the intersection of all feebly regular closed sets containing A and the feebly regular interior of A (briefly F.reg.int(A)) is the union of all feebly regular open sets contained in A.

Remark 3.6: The complement of feebly regular open set is feebly regular closed.

Theorem 3.7: If A and B are F.reg.closed sets then $A \cup B$ is F.reg.closed.

Proof: Given A and B are F.reg.closed. We know that A = f.cl(f.int(A)) and B = f.cl(f.int(A)). Now $A \cup B$ = f.cl(f.int($A \cup B$)). Thus $A \cup B$ is F.reg.closed.

Corrollary 3.8: If A and B are F.reg.open then $A \cap B$ isF.reg.open.

Theorem 3.9: Let A be a subset of X. For an element $x \in X$, $x \in$ F.reg.closed set of A if and only if $U \cap A \neq \varphi$ for every F.reg.open set U containing x.

Proof: Necessity. Let $x \in$ F.reg.closed(A). Suppose there exists F.reg.open set U containing x such that $U \cap A = \varphi$. Then $A \subset U^c$. Since U^c is F.reg.closed set containing A, we have F.reg.closed(A) $\subset U^c$, which implies that $x \notin$ F.reg.closed(A). This is a contradiction. Sufficiency. Suppose that $x \in$ F.reg.closed(A). Then by definition, there exist an F.reg.closed set F containing A such that $x \notin F$. Then $x \in F^c$ and F^c is a F.reg.open set. Also $F^c \cap A = \varphi$ which is contrary to the hypothesis. Therefore $x \in$ F.reg.closed(A).

Theorem 3.10: For any two subsets A and B of (X, τ),

(i) If $A \subset B$, then F.reg.int(A) $\subset$ F.reg.int(B)

(ii) F.reg.int($A \cap B$) = F.reg.int(A) $\cap$ F.reg.int(B)

(iii) F.reg.int(A) $\cup$ F.reg.int(B) $\subset$ F.reg.int($A \cup B$)

(iv) F.reg.int(X) = X

(v) F.reg.int(φ) = φ

Proof: (i) Let A and B be subsets of X such that $A \subset B$. Let $x \in$ F.reg.int(A). Then there exists a F.reg.open set U such that $x \in U \subset B$ and hence $x \in$ F.reg.int (B). Hence, F.reg.int(A) $\subset$F.reg.int(B).

(ii) We Know that $A \cap B \subset A$ and $A \cap B \subset B$. We have by (i) F.reg.int($A \cap B$) $\subset$F.reg.int(A) and F.reg.int ($A \cap B$)$\subset$ F.reg.int(B).

This implies that F.reg.int ($A \cap B$) $\subset$F.reg.int(A) $\cap$ F.reg.int(B) -------→(1).

Again, let $x \in$F.reg.int(A) $\cap$ F.reg.int(B). Then $x \in$F.reg.int(A) and $x \in$F.reg.int(B). Then there exists F.reg.open sets U and V such that $x \in U \subset A$ and $x \in U \subset B$. By corollary 3.8, $U \cap V$ is a F.reg.open set such that $x \in (U \cap V) \subset (A \cap B)$. Hence $x \in$F.reg.int($A \cap B$). Thus $x \in$F.reg.int(A)$\cap$ F.reg.int(B) implies that $x \in$ F.reg.int($A \cap B$). Therefore, F.reg.int (A)$\cap$F.reg.int(B) $\subset$ F.reg.int ($A \cap B$) -----→(2). From (1) and (2) , it follows that F.reg.int ($A \cap B$) = F.reg.int(A) $\cap$F.reg.int(B). The proofs of (iii), (iv) and (v) are obivious.

Lemma 3.11: LetA be a subset of X**. (i)** $(\text{F.reg.int}(A))^c$ =F.reg.cl(A^c) **(ii)** $(\text{F.reg.cl}(A))^c$ = F.reg.int(A^c).

Proof: (i) Let $x \in (\text{F.reg.int}(A))^c$. Then $x \notin$F.reg.int(A). That is , every F.reg.open set U containing x such that $U \cap A^c \neq \varphi$.

By theorem3.9, $x \in \text{F.reg.cl}(A)^c$. Therefore, $(\text{F.reg.int}(A))^c \subset \text{F.reg.cl}(A)^c$. Conversely, let $x \in$F.reg.cl(A^c). Then by theorem 3.9, $U \cap A^c \neq \varphi$ for every F.reg.open set U containing x. That is, every F.reg.open set U containing x such that

$U \not\subset A$. This implies $x \notin$F.reg.int(A). Thatis , $x \in (\text{F.reg.int}(A))^c$.

(ii) . This follows by replacing A by A^c in (i).

4. ALMOST FEEBLY TOTALLY OPEN MAPPINGS

In this section, the main results of this paper is introduced.

Definition 4.1 [7]: Let f: X→Y is said to be almost open (resp. almost closed) if the image of each regular open (resp. regular closed) set in X is open (resp. closed) in Y.

Definition 4.2: A function f : X→Y is said to be

(i) Almost feebly open (briefly almost f.open) if the image of each F.reg.open set in X is feebly open in Y.

(ii) Almost feebly closed (briefly almost f.closed) if the image of each F.reg.closed set in X is f.closed in Y.

(iii) Almost feebly clopen (briefly almost f.clopen) if the image of each F.reg.clopen set in X is feebly clopen in Y.

(iv) Feebly totally open if the image of each feebly open set in X is feebly clopen in Y.

(v) Feebly totally closed if the image of each feebly closed set in X is feebly clopen in Y.

(vi) Almost feebly totally open if the image of each feebly regular open set in X is feebly clopen in Y.

(vii) Almost feebly totally closed if the image of each feebly regular closed set in X is feebly clopen in Y.

(viii) Almost feebly totally clopen if the image of each feebly regular clopen set in X is feebly clopen in Y.

Theorem 4.3: Every almost feebly totally closed set is almost feebly closed.

Proof: Let X and Y be topological spaces. Let f : X→Y be a almost feebly totally closed mapping. To prove f is almost feebly closed, let H be any feebly regular closed subset of X. Since f is almost feebly totally closed mapping,f(H) is feebly clopen in Y. This implies that f(H) is feebly closed in Y. Therefore f is almost feebly closed.

Corrollary 4.4: Every feebly totally open set is almost feebly open.

Theorem 4.5: If a bijective function f : X→Y is almost feebly totally open, then the image of each feebly regular closed set in X is feebly clopen in Y.

Proof: Let F be a feebly regular closed set in X. Then X-F is feebly regular open in X. Since f is almost feebly totally open, f(X-F) = Y-f(F) is feebly clopen in Y. This implies that f(F) is feebly clopen in Y.

Theorem 4.6: A surjective function f : X→Y is almost feebly totally open if and only if for each subset B of Y and for each feebly regular open set U containing $f^{-1}(B)$, there is a feebly clopen set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof: Suppose $f : X \to Y$ is a surjective almost feebly totally open function and $B \subset Y$. Let U be feebly regular open set of X such that $f^{-1}(B) \subset U$. Then $V = Y - f(X-U)$ is feebly clopen subset of Y containing B such that $f^{-1}(V) \subset U$.

Theorem 4.7: A map $f: (X,\tau) \to (Y,\sigma)$ is almost feebly totally open if and only if for each subset A of (Y,σ) and each feebly regular closed set U containing $f^{-1}(A)$ there is a feebly clopen set V of Y such that $A \subset V$ and $f^{-1}(V) \subset U$.

Proof: Suppose f is almost feebly totally open. Let $A \subset Y$ and U be a feebly regular closed set of X such that $f^{-1}(A) \subset U$. Now X-U is feebly totally open. Since f is almost feebly totally open, f(X-U) is feebly clopen set in (Y,σ). Then $V = Y - f(X-U)$ is a feebly clopen set in (Y,σ). Note that $f^{-1}(A) \subset U$ implies $A \subset V$ and $f^{-1}(V) = X - f^{-1}(f(X-U)) \subset X-(X-U) = U$. That is $f^{-1}(V) \subset U$. For the converse, let F be a feebly regular open set of (X,τ). Then $f^{-1}(f(F)^c) \subset F^c$ and F^c is feebly regular closed set in (X,τ). By hypothesis, there exist a feebly clopen set V in (Y,σ) such that $f(F^c) \subset V$ and $f^{-1}(V) \subset F^c$ and so $F \subset (f^{-1}(V))^c$. Hence $V^c \subset f(F) \subset f((f^{-1}(V))^c) \subset V^c$ which implies $f(V) \subset V^c$. Since V^c is feebly clopen, f(F) is feebly clopen. That is f(F) is feebly clopen in (Y,σ). Therefore f is almost feebly totally open.

Corrollary 4.8: A map $f: (X,\tau) \to (Y,\sigma)$ is almost feebly totally closed if and only if for each subset A of (Y,σ) and each feebly regular open set U containing $f^{-1}(A)$, there is a feebly clopen set V of Y such that $A \subset V$ and $f^{-1}(V) \subset U$.

Theorem 4.9: If $f : X \to Y$ is almost feebly totally closed and A is feebly regular closed subset of X then $f/A : (A,\tau/A) \to (Y,\sigma)$ is almost feebly totally closed.

Proof: Consider the function $f/A : A \to Y$ and let V be any feebly clopen set in Y. Since f is almost feebly totally closed, $f^{-1}(V)$ is feebly regular closed subset of X. Since A is feebly regular closed subset of X and $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is feebly regular closed in A, it follows $(f/A)^{-1}(V)$ is feebly regular closed in A. Hence f/A is almost feebly totally closed.

Remark 4.10: Almost feebly totally clopen mapping is almost feebly open and almost feebly totally closed map.

5. ALMOST FEEBLY TOTALLY CONTINUOUS FUNCTIONS

In this section, some new continuous functions are introduced and discussed their characterizations.

Definition 5.1: A map $f: X \to Y$ is said to be

(i) Feebly totally continuous if $f^{-1}(V)$ is feebly clopen in X for each feebly open set V in Y.
(ii) Almost feebly totally continuous if $f^{-1}(V)$ is feebly clopen in X for each feebly regular open set V in Y.
(iii) Almost feebly totally clopen continuous if $f^{-1}(V)$ is feebly clopen in X for each feebly regular clopen set V in Y.

Theorem 5.2: A function $f : X \to Y$ is almost feebly totally continuous function if the inverse image of every feebly regular open set of Y is feebly clopen in X.

Proof: Let F be any feebly regular closed set in Y. Then Y-F is feebly regular open set in Y. By definition, $f^{-1}(Y-F)$ is feebly clopen in X. That is $X-f^{-1}(F)$ is feebly clopen in X. This implies that $f^{-1}(F)$ is feebly clopen in X.

Theorem 5.3: Every almost feebly totally continuous function is a almost feebly continuous function.
Proof: Suppose $f : X \to Y$ is almost feebly totally continuous and U is any feebly regular open subset of Y. It follows $f^{-1}(U)$ is feebly clopen in X. This implies that $f^{-1}(U)$ is feebly open in X. Therefore the function f is almost feebly continuous.

Theorem 5.4: Let X and Y be any two topological spaces, $X = A \cup B$ where A and B are feebly clopen subset of X and let $f: (X,\tau) \to (Y,\sigma)$ be a function such that f/A and f/B are almost feebly totally clopen continuous function. Then f is almost feebly totally clopen continuous function.

Proof: Let U be any feebly regular clopen in Y such that $f^{-1}(U) \neq \varphi$. Then $(f/A)^{-1}(U) \neq \varphi$ or $(f/B)^{-1}(U) \neq \varphi$ or both $(f/A)^{-1}(U) \neq \varphi$ and $(f/B)^{-1}(U) \neq \varphi$.

Case i: Suppose $(f/A)^{-1}(U) \neq \varphi$. Since f/A is almost feebly totally clopen continuous, there exist a feebly clopen set V in A such that $V \neq \varphi$ and $V \subset (f/A)^{-1}(U) \subset f^{-1}(U)$. Since V is feebly clopen in A and A is feebly clopen in X, we have V is feebly clopen in X. Thus f is almost feebly totally clopen continuous function.

Case ii: Suppose $(f/B)^{-1}(U) \neq \varphi$. Since f/B is almost feebly totally clopen continuous function, there exists a feebly clopen set V in B such that $V \neq \varphi$ and $V \subset (f/B)^{-1}(U) \subseteq f^{-1}(U)$. Since V is feebly clopen in B and B is feebly clopen in X, V is feebly clopen in X. Thus f is almost feebly totally clopen continuous function.
Case iii: Suppose $(f/A)^{-1}(U) \neq \varphi$ and $(f/B)^{-1}(U) \neq \varphi$.This follows from both the cases (i) and (ii).Thus f is almost feebly totally clopen continuous function.

Theorem 5.5: For any bijective map $f : X \rightarrow Y$ the following statements are equivalent:

(i) $f^{-1} : Y \rightarrow X$ is almost feebly totally continuous.

(ii) f is almost feebly totally open.

(iii) f is almost feebly totally closed.

Proof: (i) $\Rightarrow$ (ii): Let U be a feebly regular open set of (X,τ). By assumption,$(f^{-1})^{-1}(U) = f(U)$ is feebly clopen in (Y,σ) and so f is almost feebly totally open.

(ii) $\Rightarrow$ (iii): Let F be a feebly regular closed set of (X,τ). Then F^c is feebly regular open set in (X,τ). By assumption $f(F^c)$ is feebly clopen in (Y,σ). Hence f is almost feebly totally closed.

(iii)$\Rightarrow$(i) : Let F be a feebly regular closed set of (X,τ). By assumption, f(F) is feebly clopen set in (Y,σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore $f^{-1}$is almost feebly totally continuous.

6. SUPER FEEBLY CLOPEN CONTINUOUS FUNCTIONS

Definition 6.1: A map $f : X \rightarrow Y$ is said to be super feebly clopen continuous if for each $x \in X$ and for each feebly clopen set V containing f(x) in Y, there exist a feebly regular open set U containing x such that $f(U) \subset V$.

Theorem 6.2: Let $f : X \rightarrow Y$ be almost feebly totally open. Then f is super feebly clopen continuous if f(x) is feebly clopen in Y.

Proof: Let G be feebly clopen in Y. Now $f^{-1}(G)$ is feebly regular open in X, $f(f^{-1}(G)) = G \cap f(x)$ is feebly clopen in Y, since the intersection of feebly clopen set is feebly clopen in Y. Therefore by the definition 4.2 (vi), $f^{-1}(G)$ is feebly regular open in X. Hence f is super feebly clopen continuous function.

Theorem 6.3: If $f : X \rightarrow Y$ is surjective and almost feebly totally open, then f is super feebly clopen continuous.

Proof: Let G be feebly clopen in Y. Take $A = f^{-1}(G)$. Since f(A) = G is feebly clopen in Y, by the above theorem 6.2, A is feebly regular open in X. Therefore f is super feebly clopen continuous.

Theorem 6.4 : Let (X,τ), (Y,σ) and (Z,η) be topological spaces. Then the composition $g \circ f : (X,\tau) \rightarrow (Z,\eta)$ is super feebly clopen continuous function where $f:(X,\tau)\rightarrow(Y,\sigma)$ is super feebly clopen continuous function and $g:(Y,\sigma)\rightarrow(Z,\eta)$ is feebly clopen irresolute function.

Proof: Let A be a feebly regular closed set of (X,τ). Since f is super feebly clopen continuous, f(A) is feebly clopen in (Y,σ). Then by hypothesis f(A) is feebly clopen set. Since g is feebly clopen irresolute, $g(f(A)) = (g \circ f)(A)$. Therefore $g \circ f$ is super feebly clopen continuous.

Theorem 6.5: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ and $g : (Y,\sigma) \rightarrow (Z,\eta)$ be two mappings such that their composition $g \circ f : X \rightarrow Z$ be almost feebly totally closed mapping then the following statements are true.

(i) If f is super feebly clopen continuous and surjective, then g is feebly clopen irresolute function.

(ii) If g is feebly clopen irresolute function and injective, then f is almost feebly totally closed function.

Proof : (i) Let A be a feebly clopen set of (Y,σ). Since f is super feebly clopen continuous, $f^{-1}(A)$ is feebly regular closed in X. Since $(g \circ f)(f^{-1}(A))$ is feebly clopen in (Z,η). That is g(A) is feebly clopen in (Z,η), since f is surjective. Therefore g is feebly clopen irresolute function.

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(ii) Let B be feebly regular closed set of (X,τ). Since g ∘f is almost feebly totally closed, (g ∘f) (B) is feebly clopen set in (Z,η), since g is feebly clopen irresolute function. Now g^{-1} ((g ∘f) (B)) is feebly clopen set in (Y,σ). That is f (B) is feebly clopen set in (Y,σ). Since f is injective and therefore f is almost feebly closed function.

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